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# AN ECONOMIC MODEL FOR DETERMINING THE PRODUCTION FUNCTION FOR WATER IN AGRICULTURE

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# ACKNOWLEDGMENTS

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# AN ECONOMIC MODEL FOR DETERMINING THE PRODUCTION FUNCTION FOR WATER IN AGRICULTURE[1]

by

Christoph Beringer[2]

The years since World War II have seen the development of close cooperation between physical scientists and agricultural economists aimed at determining the shape of production functions for various factors of production used in agriculture. The work with fertilizer and, in the field of animal nutrition, the exploration of response surfaces for various combinations of feedstuffs are the most widely known examples.

To date, no such cooperative work has been reported in which the production factor "water" was the principal experimental variable, although several authors have recently presented methods of incorporating rainfall and residual soil moisture variation into fertilizer response studies.[3]

Agronomists have, of course, independently carried out numerous water response studies over the past 60 years. As physical scientists, they have usually directed their experiments towards the solution of measurement problems rather than towards the applicability of the results for purposes of economic policy.

The need for input-output experiments to ascertain more precisely economically efficient water use rates is becoming increasingly acute with the growing

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[1] This report is based on research undertaken while Dr. Beringer was in the Department of Agricultural Economics, University of California, Berkeley, during 1958-59. The work was performed under Project 1492 of the California Agricultural Experiment Station.

[2] Professor of Economics and Director of the Board of Economics Inquiry, Peshawar University, Peshawar, Pakistan.

[3] See in this connection: Jack L. Knetsch, "Moisture Uncertainties and Fertility Response Studies," Journal of Farm Economics, vol. 41, no. 1, February, 1959, pp. 70-76. See also: Frank Orazem and Roy B. Herring, "Economic Aspects of the Effects of Fertilizers, Soil Moisture and Rainfall on the Yields of Grain Sorghum in the 'Sandy Lands' of Southwest Kansas," Journal of Farm Economics, vol. 40, no. 3, August, 1958, pp. 687-708.

scarcity of this most basic resource in the large arid areas of the United States and elsewhere. In California, where a $1.75 billion bond issue recently has been approved for projects which will make additional water resources available to the southern part of the state, the need for economizing principles in the use of water resources is particularly underlined, for, under this program, an acre-foot of water may cost as much as $50 by the time it reaches its destination.

Usually, the initial reaction of economists confronted with the problem of determining a water response function in agriculture is to set up a series of experiments in which water quantity is the independent variable and crop output the dependent variable. This has been the approach used by economists in almost all production function studies carried out to date. It appears to be applicable in all those instances in which either the distribution of the input factor over the production period may be fixed on technological grounds or the storage capacity of soils can be assumed to be such that variations in the distribution of nutrients will not materially affect total output,as in the case of most fertilizers.

It is the purpose of this paper to show that the "water quantity" approach alone only has limited value in establishing a meaningful water response function. It is true that agronomists in the early part of this century used water quantity as the only causal variable when conducting experiments with water, but this approach has since been completely abandoned.[1] One reason seems to be that the water quantity approach fails to consider distribution over the irrigation cycle as an important variable. Furthermore, the results of such experiments are applicable only for the particular soil type used and under the same climatic conditions existing when the experiment was originally carried out. In recent years, soil scientists have endeavored to work out water-input crop-output relationships which are sustained irrespective of soil type and are scarcity of water has been resolved in the large urban areas of the United States and elsewhere. In California, where a $1.7 billion bond issue recently has been approved for projects which will make additional water supplies available in the southern part of the state, the need for economizing techniques in the use of water resources is particularly acute. Yet, under this project, an acre-foot of water may cost as much as $50 by the time it reaches its destination.

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[1] For studies illustrating irrigation water experiments carried out in the early part of this century, see: S. H. Beckett and H. R. Huberty, Irrigation Investigations with Field Crops at Davis and at Delhi, California, 1909-1925, California Agricultural Experiment Station Bul. 450 (Berkeley, 1928). Also see: John A. Widtsoe and L. A. Merrill, The Yields of Crops with Different Quantities of Irrigation Water, Utah Agricultural Experiment Station Bul. 117 (Logan, 1912).

Similarly, the initial reaction of economists concerned with the problem of determining a water-response function in irrigation is to rely upon a series of experiments in which water quantity is the independent variable and crop output the dependent variable. This has been the classic approach to almost all production function studies in the past. It suggests an applicability to all cases inasmuch as diminishing marginal returns to the input factor over the production period may be taken as fundamental. Changes in the average capacity of soils to hold moisture and the seasonal variations in the distribution of precipitation will not necessarily affect total capacity in the case of such experiments.

It is the purpose of this paper to show that the "water quantity" approach alone only has limited value in establishing a meaningful water-response function. It is true that agronomists in the early part of this century often made quantity the only water variable when conducting experiments with water, but this approach has since been completely abandoned.[1] The reason for this is that the water quantity approach fails to consider differences in time of application, which is an important variable. Furthermore, the results of such experiments are applicable only for the particular soil type and the area under the same climatic conditions existing when the experiment was originally conducted. Increasingly, soil scientists have endeavored to find the soil-plant-atmosphere relationships which had been lacking in experiments with crop and water.

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[1] For studies illustrating irrigation water experiments carried out in the early part of this century, see: S. H. Beckett and R. D. Robertson, Irrigation Investigations with Field Crops at Davis and at Delhi, California, 1909-1917 (California Agricultural Experiment Station Bul. 280 (Berkeley, 1917)); also: John A. Widtsoe and L. A. Merrill, The Yields of Crops with Different Quantities of Irrigation Water, Utah Agricultural Experiment Station Bul. 117 (Logan, 1912).

therefore applicable under a much wider range of conditions. Details of this development are discussed below.

## Agronomic Theories Affecting the Design of Water-Input Crop-Output Experiments

Before it is possible to suggest a workable model for a production function showing the relationship between water input and crop output, it is necessary to review briefly some of the more important findings made by agronomists and soil physicists regarding soil-water plant relationships, particularly the pattern of water extraction by plants and the availability of water to plants from various soil types. The aim of this review is to emphasize those technical considerations which are indispensable for a meaningful economic analysis of the problem. No attempt is made to do full justice to all the technical details which necessarily surround every one of the various agronomic theories.

### The Determinants of Water-Holding Capacity for Various Soil Types

The clay content of a particular soil is the most important determinant of its water-holding capacity. If the percentage of clay particles is high, as in clays and heavy loams, then the water-holding capacity will also be high; light soils (sands and sandy loams) usually have a much lower water-holding capacity.1/ The term "field capacity"2/ has been introduced to define

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1/ In addition to soil texture and clay type, there are other factors influencing the water-holding capacity of soils. W. L. Parks in "Methodological Problems in Agronomic Research Involving Fertilizer and Moisture Variables," Methodological Procedures in the Economic Analysis of Fertilizer Use Data (Ames: Iowa State College Press, 1956), p. 115, lists them in the following order:

1. Organic Matter
2. Osmotic Effects
3. Total Pore Size and Pore Size Distribution
4. Depth of Soil Profile

There are some rare instances, therefore, when a sandy soil may actually have a greater water-holding capacity than a heavy soil due to the influence of factors other than soil texture and clay type. D. W. Thorne and H. B. Peterson, Irrigated Soils (2d ed.; New York: The Blakiston Company, Inc., 1954), p. 48.

2/ For a more detailed explanation of this concept, see: Thorne and Peterson, *ibid*., p. 34.

[illegible] under a [illegible] range of [illegible]. Details of this development are discussed below.

## [illegible] Theories Affecting the [illegible] [illegible]

As an aid in [illegible], it [illegible] a production function showing the [illegible] water input and [illegible] of [illegible] of the [illegible] soil [illegible] relationships [illegible] pattern of water consumption by plants and the [illegible] of [illegible] soil types. The aim of this [illegible] and [illegible] which are [illegible] of the problem. No attempt is made to [illegible] all the [illegible] but [illegible] recent [illegible] in the [illegible].

[illegible] Determination of Water-Holding Capacity [illegible]

The [illegible] content of a [illegible] soil is [illegible] of the water-holding capacity. As the percentage of clay particles [illegible] as in clays and heavy loams, the water-holding capacity will [illegible] than [illegible] (sands and sandy loams) [illegible] holding capacity.[1] The term "field capacity" has been [illegible] and measure the maximum amount of water a soil will hold against gravity when allowed to drain freely. This point is reached in a deep, well-drained soil about 48 hours after irrigation is completed.

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[1] In addition to soil texture and clay types there are other factors that affect the water-holding capacity of soils. [illegible] in [illegible] lists the following factors:

1. Organic matter
2. [illegible]
3. [illegible] and Pore Size Distribution
4. [illegible] Soil Profile

These are [illegible] soils [illegible] have a greater water-holding capacity [illegible] D. W. Thorne and H. B. Peterson, *Irrigated Soils* (2d ed.; New York: The Blakiston Company, Inc., [illegible]), [illegible].

[2] For a more [illegible] discussion of this concept, see [illegible] 1951, [illegible].

## The Pattern of Water Extraction by Plants

The clay particles in a soil which has been wetted to field capacity can be pictured as being surrounded by a film of water, itself consisting of a large number of water particles. Water particles close to the surface of the clay particles are held relatively tightly and those further removed from that surface relatively loosely, yet with a force which must at least equal the pull of gravity. The concept of "moisture tension" (also called moisture stress or moisture suction) has been introduced to measure the force with which water particles are held by a particular soil at any one time.[1/] As plant roots begin to remove soil moisture, they first absorb those water particles which are relatively loosely attached to the surface of the clay particles. Moisture tension increases after these loosely attached water particles have been removed; that is, plants must begin to extract water which is held more tightly. As this process continues, a point called the "wilting point," or "permanent wilting percentage" (PWP), is reached, where the amount of water that plants are able to extract is exactly the same as the amount they lose through transpiration.[2/] Finally, the power of roots to extract moisture becomes less than the power of clay particles to retain it, and the plant gradually dies.

In general, soils with a large proportion of clay particles (clays and heavy loams), representing a greater surface area per volume of soil, will hold a greater total quantity of water at a given moisture tension than light soils (sands and sandy loams). Furthermore, the range of readily available moisture, which is the same as the difference between field capacity and permanent wilting point, is relatively wide for clays and relatively narrow for sandy soils. Other factors remaining the same, moisture tension will increase more rapidly

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[1/] See in this connection: L. A. Richards and L. R. Weaver, "Moisture Retention by Some Irrigated Soils as Related to Soil Moisture Tension," Journal of Agricultural Research, vol. 69, no. 6, December 15, 1944, pp. 215-235.

[2/] See in this connection: L. J. Briggs and H. L. Schantz, The Wilting Coefficient for Different Plants and Its Indirect Determination, U. S. Bureau of Plant Industry Bulletin No. 230 (Washington, 1912).

in light soils than in heavy soils; the former, therefore, will generally require more frequent irrigations than the latter.

The idea of relating plant growth first to moisture tension and only later and indirectly to water quantity has been a great step forward in agronomic research with water. As a result, a more widely applicable general relationship between moisture conditions and plant growth has been developed. The introduction of the concept of moisture tension has made it possible to construct production functions which are, in a sense, independent of soil type and water quantity. Once this general relationship is known, it is relatively easy to determine, on the basis of known moisture extraction curves, the optimum irrigation frequency as well as the quantity of water necessary to fill a given volume of various soil types to field capacity at each irrigation. An example of a moisture extraction curve for a particular soil type is given in Figure 1. It shows how the tension in a Panoche loam increases as the percentage of soil moisture is reduced.

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## The Pattern of Water Absorption by Soils After an Irrigation

In addition to the pattern of water extraction by plants, the way in which water enters a soil mass is also of considerable significance in determining a crop response function to water.

In this connection, it has been pointed out by Kramer,[1] in reviewing the literature which deals with the experimental control of soil moisture, that it is impossible to wet any soil mass to less than its field capacity. In particular, he refers to the work of Veihmeyer[2] and Schantz,[3] who came to the conclusion that, if a small quantity of water is applied to a mass of dry soil, the uppermost layer is filled to field capacity while the rest of the soil remains unaffected. This phenomenon is explained by the fact that, as long as there are soil particles in the upper layer of soil whose moisture-holding

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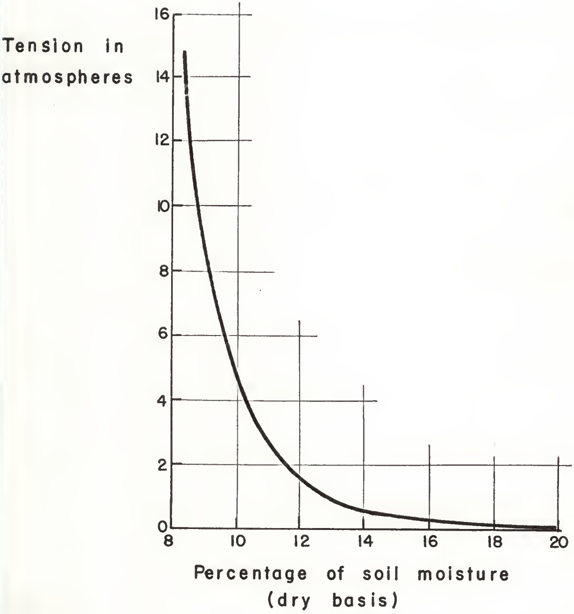
[1] Paul J. Kramer, Plant and Soil Water Relationships (New York: McGraw-Hill Book Co., 1949), p. 94.

[2] F. J. Veihmeyer, "Some Factors Affecting the Irrigation Requirements of Deciduous Orchards," Hilgardia, vol. 2, no. 6, January, 1927, pp. 125-184.

[3] Schantz, "Soil Moisture in Relation to the Growth of Plants," Journal of the American Society of Agronomy, vol. 17, November, 1925, pp. 705-711.

FIGURE 1

The Relationship Between Soil Moisture Content and Tension in Panoche Loam



Source: Cecil H. Wadleigh, "Soil Moisture in Relation to Plant Growth," Water, The Yearbook of Agriculture, 1955 (Washington: U. S. Government Printing Office, 1955), p. 359.

capacity is not yet exhausted, little or no moisture will be released to deeper soil layers. As more moisture is added, the soil is wetted to greater depth, but the wet layer above will always be separated from the dry layer below by a very sharp line of demarcation. These considerations lead irrigation scientists to the conclusion that there is no satisfactory way to control the moisture content of any volume of soil below its field capacity. The moisture content of any particular layer of soil can be reduced only through extraction by plants or through evaporation.

With respect to the shape of the crop-response curve to water quantity, these considerations are important because they suggest the following: If a small amount of water is applied during each of a fixed number of irrigations, only the uppermost layer of the soil will be wetted. Germination, root development, and plant growth will be restricted to this layer, and the resulting yield response will be zero or, at best, a very small amount. As more water is applied, a second layer of soil will be wetted; germination and root development will be improved--and so will the resulting production. As this process is carried on, it will result in a production-response curve similar to that suggested for fertilizer inputs by Mitscherlich1/ and Spillman.2/ Such a curve will approximate our usual concept of the law of diminishing returns, although it may do this only in a discrete form because of experimental difficulties.3/ The crop-response curve understood in this context appears to have been the basis for most of the empirical work carried out in the early part of this century.

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1/ E. O. Mitscherlich, "Das Gesetz des Minimums und das Gesetz des Abnehmenden Bodenertrags," Landwirtschaftliches Jahrbuch, vol. 38, 1909, pp. 537-552.

2/ W. J. Spillman, Use of Exponential Yield Curves in Fertilizer Experiments, U. S. Department of Agriculture Technical Bulletin No. 348 (Washington, April, 1933), 67p.

3/ This argument implies, of course, a ceteris paribus assumption and does not mean that plant response is always proportional to the depth to which a soil is wetted. It has been pointed out to me by D. W. Thorne that studies both here and abroad have shown repeatedly that yields can be very high if only a shallow amount of soil is kept wet, provided that there is enough fertility in that shallow layer of soil.

The existence of a law of diminishing returns with respect to water inputs, as demonstrated here, does not in itself provide a sound theoretical and practical basis for experimentation. Among other things, the fact that the size of a plant's root zone is largely determined genetically is completely ignored. This fact alone makes it unrealistic to divorce the water quantity problem from distributional aspects. When crops are deep-rooted and only a small quantity of water is available, it will usually be more profitable to apply all the available water at once and thus wet a large part of the root zone than to distribute it over many irrigations. This aspect of the problem will be discussed further in subsequent paragraphs.

### Two Schools of Thought Regarding the Availability of Water to Plants Between Field Capacity and Wilting Point

For the reasons given in paragraphs 1, 2, and 3 above, there appears to be general agreement among agronomists that it is more desirable to work out production functions in response to variations in moisture tension than to use water quantity as the independent variable. In spite of the agreement on this point, there remains a significant difference of opinion regarding the pattern of water extraction by crops between field capacity and the wilting point. It will be shown that the type of water experiments conducted to establish production functions for water is determined by the school of thought which has been adopted.

One school of thought, associated mainly with Veihmeyer, Hendrickson, and Adams at California, maintains that, in the area between field capacity and PWP, plants extract the soil moisture equally well for their continued growth.1/ This theory is supported by the fact that the force with which water is held by soil particles changes relatively little between these two critical points;

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1/ See in this connection: Frank Adams, Cotton Irrigation Investigations in San Joaquin Valley, California, 1926-1935, California Agricultural Experiment Station Bul. 668 (Berkeley, 1942), 93p.; A. H. Hendrickson and Veihmeyer, Irrigation Experiments with Peaches in California, California Agricultural Experiment Station Bul. 479 (Berkeley, 1929); Hendrickson and Veihmeyer, Irrigation Experiments with Prunes, California Agricultural Experiment Station Bul. 573 (Berkeley, 1934), 44p; Hendrickson and Veihmeyer, "Irrigation Experiments with Grapes," Proceedings, American Society of Horticultural Science, vol. 28, 1931, pp. 151-157; Hendrickson and Veihmeyer, "Responses of Fruit Trees to Comparatively Large Amounts of Available Moisture," Proceedings, American Society of Horticultural Science, vol. 35, 1937, pp. 289-292.

in particular, it is argued that the soil moisture tension in most soils does not exceed 1 atmosphere until most of the available water is removed.[1/] For example, a glance at Figure 1 shows for a particular soil type that between 20 per cent and 13 per cent of soil moisture, the tension rises by only 1 atmosphere, whereas between 13 per cent and 8 per cent, the tension rises by 15 atmosphere. If this theory holds, it implies that it is likely to be unnecessary to reirrigate a given soil mass until the soil moisture has been reduced to the wilting point. Furthermore, this theory has given rise to the belief that in the case of water inputs, the law of diminishing returns may not be applicable in the same way it is for other productive inputs.[2/] Translated into terms familiar to economists, the Veihmeyer theory means that the marginal physical product of irrigation frequencies, which would never allow the water content in a given soil volume to be reduced to the wilting point, is zero or close to zero, whereas the marginal physical product of irrigation frequencies, which permit the soil water content to be reduced significantly below the wilting percentage for any extended period of time, would be close to infinity because total crop yield would be seriously reduced. The relationship is illustrated graphically in Figure 2.

According to this theory, the optimum irrigation frequency would be that which reirrigates whenever the soil moisture percentage has been reduced to the permanent wilting percentage. Experiments based on this theory would be designed so as to find for each crop and each soil type the period of time required to reduce the soil moisture to the permanent wilting percentage.

The opposing school of thought, composed of agronomists working at various American universities, implicitly questions the Veihmeyer theory on empirical as well as on theoretical grounds.

They argue, first of all, that the soil moisture content which causes wilting in a particular soil must be expressed by a distribution rather than by one single number, because the moisture content of a given soil volume does

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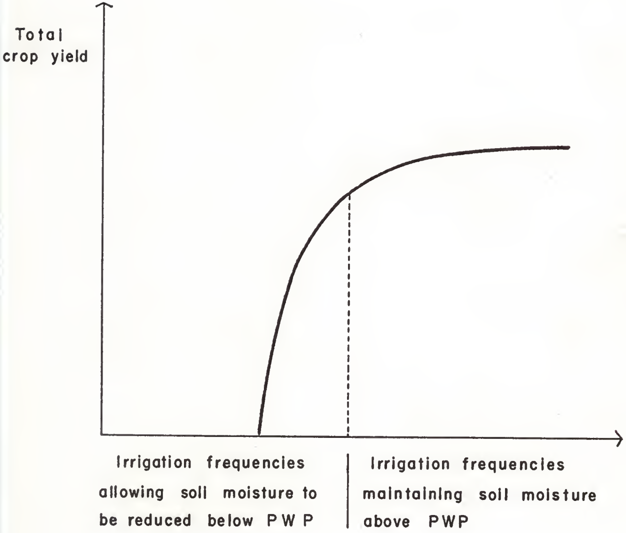
[1/] C. H. Wadleigh, "The Integrated Soil Moisture Stress Upon a Root System in a Large Container of Saline Soil," Soil Science, vol. 61, 1946, p. 225.

[2/] For an explicit statement of this interpretation, see: Ivan M. Lee, Optimum Water Resource Development: A Preliminary Statement of Methodology for Quantitative Analysis, University of California, Giannini Foundation Mimeographed Report No. 206 (Berkeley, July, 1958), p. 25.

FIGURE 2

Effect of Irrigation Frequencies on Total Crop Yield
According to the Veihmeyer Theory



not change uniformly after an irrigation.[1] Two reasons are given for this lack of uniformity: (1) the moisture content of the upper layers of the soil profile is reduced more quickly to the wilting percentage through evaporation, while, at the same time, considerably more soil moisture remains in lower soil strata; and (2) in those parts of the soil profile which contain the heaviest concentration of plant roots, soil moisture tension will increase more rapidly, resulting in areas within the profile which contain much less available soil moisture than others. The almost complete absence of lateral or vertical movement of soil water in response to these relatively small differences in soil moisture tension, which result from plant root extraction, aggravates this problem. Richards and Wadleigh state in this connection:[2]

> "The effective distance through which water in the available range can move toward the root is certainly of the order of inches and not feet. The pattern of moisture extraction is therefore largely a matter of the active root distribution. Root distribution . . . is mainly determined by the genetic character of the plant but is modified by plant spacing as well as by soil and climatic factors."

Since the roots of a crop extend vertically through a certain part of the soil profile and since the moisture tension within this profile does not change uniformly after an irrigation, it follows that the soil moisture condition which generates wilting is not a single point but, rather, a certain distribution of soil moisture throughout the profile. More specifically, it is argued that (1) within any one layer of soil, the moisture tension will be an increasing function of time, t, and (2) at any one period of time, moisture tension within the soil profile between layers will be a decreasing function of depth, x. These relationships are illustrated schematically in Figure 3, where moisture tension is plotted on the vertical, depth of soil on the horizontal axis, and different time periods after the date of irrigation are represented by curves, $t_o$ (field capacity), . . . , $t_n$.

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[1] For a discussion of the methods used for measuring soil moisture, see: Thorne and Peterson, op. cit.

[2] Joint Committee on Soil Tilth, American Society of Agronomy, and American Society of Agricultural Engineers, Soil Physical Conditions and Plant Growth, ed. Byron T. Shaw (New York: Academic Press, Inc., 1952), vol. II, p. 85.

[illegible] 1/ [illegible]

> [illegible]

[illegible]

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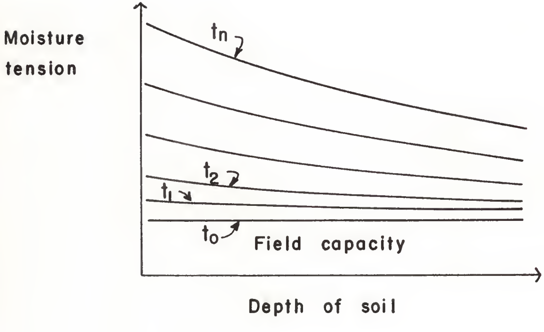
1/ [illegible]

2/ [illegible]

FIGURE 3

Schematic Illustration of the Relationship Between Soil Moisture Tension, Time, and Depth of the Soil Profile



From an analytical point of view, that is, for purposes of determining crop output as a function of various soil moisture stress conditions, it would be extremely cumbersome to work with joint distributions of this kind as independent variables. To overcome this difficulty, Wadleigh[1/] and Taylor[2/] have suggested methods of combining these distributions into a single number by means of a concept which they have called "integrated moisture stress."

The method suggested by Wadleigh has particular application to saline soils, which are very common in the western United States and many other arid parts of the world. In his system, the total soil moisture stress, S, at any one point in time is expressed as a linear function of the soil moisture tension, T, and the osmotic pressure, $\pi$, the latter resulting from the presence of salts in the soil. The relationship is expressed in Equation (1):

$$(1) \qquad S = \overset{\text{tension}}{T} + \overset{\text{osmotic pressure}}{\pi}$$

T is a decreasing function of depth, but, as a result of leaching which leads to greater salt concentrations in lower soil strata, $\pi$ is an increasing function of depth, and so the two forces tend to make the total soil moisture stress uniform throughout the soil profile. With this in mind, Wadleigh makes the assumption that, in certain saline soils, S is no longer a function of depth but only a function of time. By integrating S with respect to time, as in Equation (2),

$$(2) \qquad A = \int_{t_o}^{t_x} S_{dt}$$

he obtains the integrated atmosphere days, A; that is, the sum total of the different stresses exerted each day during the irrigation interval. $t_x$ represents the number of days in the irrigation interval, and $\frac{A}{t_x}$ represents the

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[1/] Wadleigh, op. cit., pp. 225-238.

[2/] S. A. Taylor, "Estimating the Integrated Soil Moisture Tension in the Root Zone of Growing Crops," Soil Science, vol. 73, 1952, pp. 331-340.

average soil moisture stress during the irrigation interval. Either A or $\frac{A}{t_x}$ can conceivably be used as the independent variable in the determination of a physical production function for water.

The method suggested by Taylor is similar but more general, inasmuch as it considers both time and depth variation of soil moisture tension expressed in a general form as follows:

$$T = F(x,t) \tag{3}$$

where T again denotes the total soil moisture tension, x is the depth below the surface, and t is time. In this system, it is first necessary to express soil moisture tension with respect to depth as a function of x as in Equation (4):

$$T_i = f(x) \tag{4}$$

and with respect to time as in Equation (5):

$$T_j = g(t) \tag{5}$$

After combining these two equations into (6),

$$T_{ij} = F(x,t) \tag{6}$$

the integrated soil moisture tension is expressed by the double integral

$$T_r = \int_{t_o}^{t_x} \int_{x = d_o}^{x = d} T_{ij} dxdt \tag{7}$$

where $T_r$ represents the integrated soil moisture tension in the root zone.

Both Wadleigh and Taylor show in detail how these integrals are evaluated and empirically approximated, and the reader is referred to these publications for a detailed exposition of the procedures used.

Since these methods were developed, a number of experiments have been conducted in which crop output was related to variations in integrated moisture stress. Results of some of these experiments are illustrated in Figures 4, 5, and 6.[1/] They clearly show that as the mean integrated moisture tension increases, the resulting crop yield decreases. These experiments leave little doubt about the existence of the law of diminishing returns in response to changes in integrated moisture stress--or rather its inverse[2/]--if the moisture stress is allowed to vary over a sufficiently wide range.

Experiments based on variations in integrated moisture stress generally do not show a very high correlation between the number of irrigations and the total quantity of water necessary to keep the moisture stress of a given soil volume within specified limits. This means that, as the integrated moisture stress is allowed to rise (less frequent irrigations), there will be no proportional decrease in the total water quantity used. The reason is that it takes less water per irrigation to bring a given soil volume up to field capacity if only a small variation in moisture stress is allowed than it does when the moisture content has been allowed to fall much lower. Due to unavoidable wastes and a greater amount of evapotranspiration, more frequent

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[1/] Taylor, "Use of Mean Soil Moisture Tension to Evaluate the Effect of Soil Moisture on Crop Yields," Soil Science, vol. 74, no. 3, 1952, pp. 217-226.

For additional experimental results illustrating these relationships, see: Wadleigh, H. C. Gauch, and O. C. Magistad, Growth and Rubber Accumulation in Guayule as Conditioned by Soil Salinity and Irrigation Regime, U. S. Department of Agriculture Technical Bulletin No. 925 (Washington, November, 1946), p. 34. Also: Wadleigh and A. D. Ayers, "Growth and Biochemical Composition of Bean Plants as Conditioned by Soil Moisture Tension and Salt Concentration," Plant Physiology, vol. 20, 1945, pp. 106-132.

It should be pointed out that there are some instances, as was shown recently in a number of cotton experiments in California, where the high tension treatments resulted in considerable reductions in vegetative growth but had little effect on the yield of cotton as such. See, for example: J. R. Stockton and L. D. Doneen, "Factors in Cotton Irrigation," California Agriculture, vol. 11, no. 4, April, 1957, p. 16.

[2/] It really does not matter in principle how we plot the production function. Since we expect low yields to occur as a result of high moisture tension treatments, we would have to plot yield against the inverse of moisture tension in order to produce a response curve which has the shape to which we are accustomed from the literature.

FIGURE 4

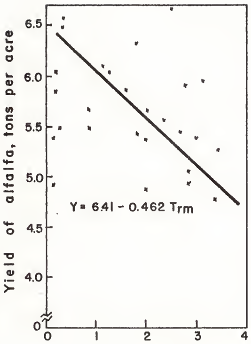
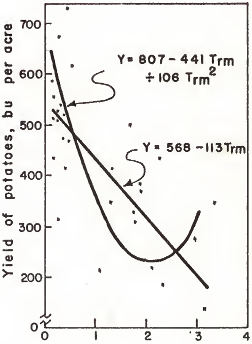
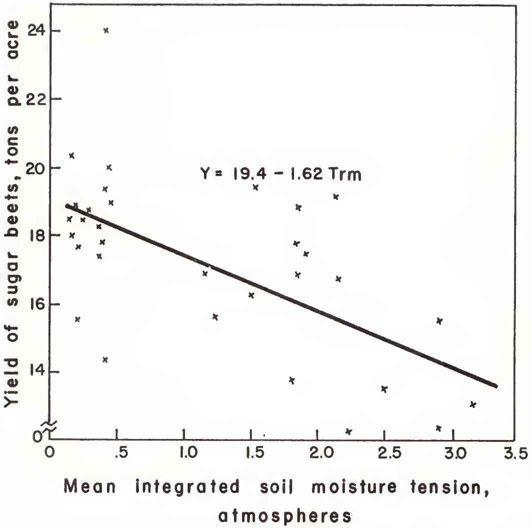


FIGURE 5



ource: S. A. Taylor, "Use of Mean Soil Moisture Tension to Evaluate the Effect of Soil Moisture on Crop Yields," Soil Science, vol. 74, no. 3, 1952, pp. 217-226.

FIGURE 6



Source: S. A. Taylor, "Use of Mean Soil Moisture Tension to Evaluate the Effect of Soil Moisture on Crop Yields," Soil Science, vol. 74, no. 3, 1952, pp. 217-226.

irrigations will, of course, require a somewhat greater total quantity of water than the less frequently irrigated plots. The data in Table 1, which were taken from an experiment with alfalfa hay conducted at Yuma, Arizona, illustrate this point. In this particular experiment, three levels of water application were used--the "wet" treatment consisting of 24 irrigations, the "medium" treatment consisting of 18 irrigations, and the "dry" treatment consisting of 13 irrigations. While the number of irrigations varied between 13 and 24, representing an increase of 85 per cent, the total quantity of water applied on the wet plots was only 25 per cent higher than that used on the dry plots. Differences in production response resulting from variations in moisture tension may therefore be due not so much to differences in the total quantity of water applied but rather to its better time distribution. The implications of this for economic analysis will be discussed below.

## Conclusions for Economic Analysis

Agronomic theory, which has been reviewed in this paper, leads to the conclusion that the law of diminishing returns with respect to water inputs may be viewed in two ways--first, in terms of the relationship between output and the use of different water quantities and, second, in terms of the relationship between output and the existence of various moisture stress conditions which are allowed to occur between irrigations.

this is not convertible to total q.

The analysis of soil-water plant relationships suggests that experiments based on the first type of production function, in which crop output is simply a function of water input quantity, are not likely to lead to results which are useful for purposes of economic policy. It is suggested, therefore, that future cooperative work between economists and agronomists be based upon the second type of production function, in which output is a nonlinear function of the inverse of various moisture stress conditions which are allowed to occur between irrigations. This relationship is expressed mathematically in its most general form in Equation (8).

## TABLE 1

The Effect of Moisture and Phosphorous Levels on Yields of Alfalfa Hay at Yuma, Arizona (Cumulative Yield for 1949, 1950, and 1951)

| Pounds $P_2O_5$ per acre | Hay yield (three years) | | | |
|---|---|---|---|---|
| | Wet | Medium | Dry | Average |
| | tons per acre | | | |
| 100 | 27.3 | 24.3 | 21.8 | 24.5 |
| 200 | 32.0 | 25.8 | 24.3 | 27.4 |
| 400 | 36.6 | 31.1 | 28.7 | 32.1 |
| 600 | 38.0 | 33.1 | 29.1 | 33.4 |
| Average | 33.6 | 27.6 | 26.0 | 20.3 |
| Average number of irrigations per year | 24 | 18 | 13 | |
| Average number of inches of water applied per year (total) | 86 | 72 | 69 | |
| Average number of inches of irrigation | 3.58 | 4.00 | 5.31 | |

Source: C. O. Stanberry, E. D. Converse, H. R. Haise, and O. J. Kelley, "Alfalfa Production on the Yuma Mesa as Affected by Moisture and Phosphorous Levels," *Proceedings, Soil Science Society of America*, vol. 19, no. 3, July, 1955.

(8) $$Y = f(S^{-1})$$

where Y equals value of crop output and S equals moisture stress conditions allowed to occur between irrigations.[1/] Differentiating (8), above, with respect to S yields a marginal value product which shows the rate of increase in total output resulting from a reduction in moisture stress conditions which are allowed to occur between irrigations.

For purposes of economic analysis, $S^{-1}$ is only an index and does not in itself have a price which could be compared with the marginal value product. Therefore, before it is possible to determine an economic optimum, it is necessary to express $S^{-1}$ as a function of total water quantity and other associated inputs. In this connection, it will be observed first that, as the maximum moisture stress conditions which are allowed to occur between irrigations decrease, the irrigation frequency rises. Associated with higher irrigation frequencies are somewhat larger total water inputs and considerably larger inputs of labor and capital. Only when the combined costs of these factors, which are associated with each value of $S^{-1}$, have been ascertained will it be possible to establish for each of various soil types and crops those irrigation frequencies which are economically optimum.

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[1/] The value for S can be determined and expressed in a number of ways:

1. On the basis of one reading of moisture stress taken at some specified point within the soil profile.
2. As the mean $\frac{\Sigma S_i}{N}$ of a number of moisture stress readings taken at various points within the profile at one time.
3. As the sum of the daily moisture stress conditions since the time of the last irrigation, $\Sigma_i S_i$, which have occurred.